Maple 2018.2 Integration Test Results
on the problems in "5 Inverse trig functions/5.6 Inverse cosecant"

Test results for the 48 problems in "5.6.1 u (a+b arccsc(c x))^n.txt"

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \,\mathrm{arccsc}(c x)}{x^3}\, dx$$

Optimal(type 3, 46 leaves, 4 steps):

$$\frac{b c^2 \,\mathrm{arccsc}(c x)}{4} + \frac{-a - b \,\mathrm{arccsc}(c x)}{2 x^2} - \frac{b c \sqrt{1 - \frac{1}{c^2 x^2}}}{4 x}$$

Result(type 3, 117 leaves):

$$-\frac{a}{2 x^2} - \frac{b \,\mathrm{arccsc}(c x)}{2 x^2} + \frac{c b \sqrt{c^2 x^2 - 1} \,\arctan\left(\frac{1}{\sqrt{c^2 x^2 - 1}}\right)}{4 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}\, x} - \frac{c b}{4 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}\, x} + \frac{b}{4 c \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}\, x^3}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int x^3 \left(a + b \,\mathrm{arccsc}(c x)\right)^2 dx$$

Optimal(type 3, 93 leaves, 5 steps):

$$\frac{b^2 x^2}{12 c^2} + \frac{x^4 \left(a + b \,\mathrm{arccsc}(c x)\right)^2}{4} + \frac{b^2 \ln(x)}{3 c^4} + \frac{b x \left(a + b \,\mathrm{arccsc}(c x)\right) \sqrt{1 - \frac{1}{c^2 x^2}}}{3 c^3} + \frac{b x^3 \left(a + b \,\mathrm{arccsc}(c x)\right) \sqrt{1 - \frac{1}{c^2 x^2}}}{6 c}$$

Result(type 3, 207 leaves):

$$\frac{x^4 a^2}{4} + \frac{b^2 \,\mathrm{arccsc}(c x)^2 x^4}{4} + \frac{b^2 \,\mathrm{arccsc}(c x) \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}\, x^3}{6 c} + \frac{b^2 x^2}{12 c^2} + \frac{b^2 \,\mathrm{arccsc}(c x)\, x \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{3 c^3} - \frac{b^2 \ln\left(\frac{1}{c x}\right)}{3 c^4} + \frac{a b x^4 \,\mathrm{arccsc}(c x)}{2}$$
$$+ \frac{a b x^3}{6 c \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{a b x}{6 c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{a b}{3 c^5 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}\, x}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(a + b \,\mathrm{arccsc}(c x)\right)^2 dx$$

Optimal(type 4, 159 leaves, 8 steps):

$$\frac{b^2 x}{3 c^2}+\frac{x^3 (a+b \operatorname{arccsc}(c x))^2}{3}+\frac{2 b (a+b \operatorname{arccsc}(c x)) \operatorname{arctanh}\left(\frac{I}{c x}+\sqrt{1-\frac{1}{c^2 x^2}}\right)}{3 c^3}-\frac{I b^2 \operatorname{polylog}\left(2,-\frac{I}{c x}-\sqrt{1-\frac{1}{c^2 x^2}}\right)}{3 c^3}$$
$$+\frac{I b^2 \operatorname{polylog}\left(2,\frac{I}{c x}+\sqrt{1-\frac{1}{c^2 x^2}}\right)}{3 c^3}+\frac{b x^2 (a+b \operatorname{arccsc}(c x)) \sqrt{1-\frac{1}{c^2 x^2}}}{3 c}$$

Result(type 4, 326 leaves):

$$\frac{x^3 a^2}{3}+\frac{x^3 b^2 \operatorname{arccsc}(c x)^2}{3}+\frac{b^2 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}} \operatorname{arccsc}(c x) x^2}{3 c}+\frac{b^2 x}{3 c^2}+\frac{b^2 \operatorname{arccsc}(c x) \ln\left(1+\frac{I}{c x}+\sqrt{1-\frac{1}{c^2 x^2}}\right)}{3 c^3}-\frac{I b^2 \operatorname{polylog}\left(2,-\frac{I}{c x}-\sqrt{1-\frac{1}{c^2 x^2}}\right)}{3 c^3}$$
$$-\frac{b^2 \operatorname{arccsc}(c x) \ln\left(1-\frac{I}{c x}-\sqrt{1-\frac{1}{c^2 x^2}}\right)}{3 c^3}+\frac{I b^2 \operatorname{polylog}\left(2,\frac{I}{c x}+\sqrt{1-\frac{1}{c^2 x^2}}\right)}{3 c^3}+\frac{2 x^3 a b \operatorname{arccsc}(c x)}{3}+\frac{a b x^2}{3 c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{a b}{3 c^3 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}$$
$$+\frac{a b \sqrt{c^2 x^2-1} \ln\left(c x+\sqrt{c^2 x^2-1}\right)}{3 c^4 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}} x}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int x (a+b \operatorname{arccsc}(c x))^2 \, dx$$

Optimal(type 3, 51 leaves, 4 steps):

$$\frac{x^2 (a+b \operatorname{arccsc}(c x))^2}{2}+\frac{b^2 \ln(x)}{c^2}+\frac{b x (a+b \operatorname{arccsc}(c x)) \sqrt{1-\frac{1}{c^2 x^2}}}{c}$$

Result(type 3, 132 leaves):

$$\frac{x^2 a^2}{2}+\frac{b^2 x^2 \operatorname{arccsc}(c x)^2}{2}+\frac{b^2 \operatorname{arccsc}(c x) x \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}{c}-\frac{b^2 \ln\left(\frac{1}{c x}\right)}{c^2}+a b x^2 \operatorname{arccsc}(c x)+\frac{a b x}{c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{a b}{c^3 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}} x}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b\,\mathrm{arccsc}(c\,x))^2}{x}\,\mathrm{d}x$$

Optimal(type 4, 127 leaves, 6 steps):

$$\frac{I\,(a+b\,\mathrm{arccsc}(c\,x))^3}{3\,b} - (a+b\,\mathrm{arccsc}(c\,x))^2 \ln\left(1-\left(\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)^2\right) + I\,b\,(a+b\,\mathrm{arccsc}(c\,x))\,\mathrm{polylog}\left(2, \left(\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)^2\right)$$
$$- \frac{b^2\,\mathrm{polylog}\left(3, \left(\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)^2\right)}{2}$$

Result(type 4, 360 leaves):

$$a^2 \ln(c\,x) + \frac{I\,b^2\,\mathrm{arccsc}(c\,x)^3}{3} - b^2\,\mathrm{arccsc}(c\,x)^2 \ln\left(1+\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right) + 2\,I\,b^2\,\mathrm{arccsc}(c\,x)\,\mathrm{polylog}\left(2, -\frac{I}{c\,x}-\sqrt{1-\frac{1}{c^2\,x^2}}\right) - 2\,b^2\,\mathrm{polylog}\left(3, -\frac{I}{c\,x}\right.$$
$$\left. - \sqrt{1-\frac{1}{c^2\,x^2}}\right) - b^2\,\mathrm{arccsc}(c\,x)^2 \ln\left(1-\frac{I}{c\,x}-\sqrt{1-\frac{1}{c^2\,x^2}}\right) + 2\,I\,b^2\,\mathrm{arccsc}(c\,x)\,\mathrm{polylog}\left(2, \frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right) - 2\,b^2\,\mathrm{polylog}\left(3, \frac{I}{c\,x}\right.$$
$$\left. + \sqrt{1-\frac{1}{c^2\,x^2}}\right) + I\,a\,b\,\mathrm{arccsc}(c\,x)^2 - 2\,a\,b\,\mathrm{arccsc}(c\,x)\,\ln\left(1-\frac{I}{c\,x}-\sqrt{1-\frac{1}{c^2\,x^2}}\right) - 2\,a\,b\,\mathrm{arccsc}(c\,x)\,\ln\left(1+\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)$$
$$+ 2\,I\,a\,b\,\mathrm{polylog}\left(2, \frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right) + 2\,I\,a\,b\,\mathrm{polylog}\left(2, -\frac{I}{c\,x}-\sqrt{1-\frac{1}{c^2\,x^2}}\right)$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b\,\mathrm{arccsc}(c\,x))^2}{x^5}\,\mathrm{d}x$$

Optimal(type 3, 116 leaves, 5 steps):

$$\frac{b^2}{32\,x^4} + \frac{3\,b^2\,c^2}{32\,x^2} + \frac{3\,a\,b\,c^4\,\mathrm{arccsc}(c\,x)}{16} + \frac{3\,b^2\,c^4\,\mathrm{arccsc}(c\,x)^2}{32} - \frac{(a+b\,\mathrm{arccsc}(c\,x))^2}{4\,x^4} - \frac{b\,c\,(a+b\,\mathrm{arccsc}(c\,x))\,\sqrt{1-\frac{1}{c^2\,x^2}}}{8\,x^3}$$
$$- \frac{3\,b\,c^3\,(a+b\,\mathrm{arccsc}(c\,x))\,\sqrt{1-\frac{1}{c^2\,x^2}}}{16\,x}$$

Result(type 3, 264 leaves):

$$-\frac{a^2}{4\,x^4} - \frac{b^2\,\mathrm{arccsc}(c\,x)^2}{4\,x^4} + \frac{3\,b^2\,c^4\,\mathrm{arccsc}(c\,x)^2}{32} - \frac{3\,c^3\,b^2\,\mathrm{arccsc}(c\,x)\,\sqrt{\frac{c^2\,x^2-1}{c^2\,x^2}}}{16\,x} - \frac{c\,b^2\,\mathrm{arccsc}(c\,x)\,\sqrt{\frac{c^2\,x^2-1}{c^2\,x^2}}}{8\,x^3} + \frac{b^2}{32\,x^4} + \frac{3\,b^2\,c^2}{32\,x^2} - \frac{a\,b\,\mathrm{arccsc}(c\,x)}{2\,x^4}$$

$$+\frac{3 c^3 a b \sqrt{c^2 x^2-1}\, \arctan\left(\frac{1}{\sqrt{c^2 x^2-1}}\right)}{16 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}-\frac{3 c^3 a b}{16 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}+\frac{c a b}{16 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x^3}+\frac{a b}{8 c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x^5}$$

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Problem 11: Result more than twice size of optimal antiderivative.
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$$\int \frac{(a+b \operatorname{arccsc}(c x))^3}{x^2}\, dx$$

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Optimal(type 3, 76 leaves, 5 steps):
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$$\frac{6 b^2 (a+b \operatorname{arccsc}(c x))}{x}-\frac{(a+b \operatorname{arccsc}(c x))^3}{x}+6 b^3 c \sqrt{1-\frac{1}{c^2 x^2}}-3 b c (a+b \operatorname{arccsc}(c x))^2 \sqrt{1-\frac{1}{c^2 x^2}}$$

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Result(type 3, 198 leaves):
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$$c \left(-\frac{a^3}{c x}+b^3 \left(-\frac{\operatorname{arccsc}(c x)^3}{c x}-3 \operatorname{arccsc}(c x)^2 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}+6 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}+\frac{6 \operatorname{arccsc}(c x)}{c x}\right)+3 a b^2 \left(-\frac{\operatorname{arccsc}(c x)^2}{c x}+\frac{2}{c x}\right.\right.$$
$$\left.\left.-2 \operatorname{arccsc}(c x) \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\right)+3 a^2 b \left(-\frac{\operatorname{arccsc}(c x)}{c x}-\frac{c^2 x^2-1}{\sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, c^2 x^2}\right)\right)$$

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Problem 14: Unable to integrate problem.
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$$\int (d x)^m (a+b \operatorname{arccsc}(c x))\, dx$$

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Optimal(type 5, 62 leaves, 3 steps):
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$$\frac{(d x)^{1+m} (a+b \operatorname{arccsc}(c x))}{d (1+m)}+\frac{b (d x)^m \operatorname{hypergeom}\left(\left[\frac{1}{2},-\frac{m}{2}\right],\left[1-\frac{m}{2}\right],\frac{1}{c^2 x^2}\right)}{c m (1+m)}$$

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Result(type 8, 16 leaves):
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$$\int (d x)^m (a+b \operatorname{arccsc}(c x))\, dx$$

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Problem 15: Result more than twice size of optimal antiderivative.
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$$\int (e x+d)^2 (a+b \operatorname{arccsc}(c x))\, dx$$

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Optimal(type 3, 109 leaves, 10 steps):
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$$-\frac{b d^3 \operatorname{arccsc}(c x)}{3 e}+\frac{(e x+d)^3 (a+b \operatorname{arccsc}(c x))}{3 e}+\frac{b (6 c^2 d^2+e^2) \operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2 x^2}}\right)}{6 c^3}+\frac{b d e x \sqrt{1-\frac{1}{c^2 x^2}}}{c}+\frac{b e^2 x^2 \sqrt{1-\frac{1}{c^2 x^2}}}{6 c}$$

Result(type 3, 360 leaves):

$$\frac{a e^2 x^3}{3}+a e x^2 d+a x d^2+\frac{a d^3}{3 e}+\frac{b e^2 \operatorname{arccsc}(c x) x^3}{3}+b e \operatorname{arccsc}(c x) x^2 d+b \operatorname{arccsc}(c x) x d^2+\frac{b d^3 \operatorname{arccsc}(c x)}{3 e}-\frac{b \sqrt{c^2 x^2-1}\, d^3 \arctan\left(\frac{1}{\sqrt{c^2 x^2-1}}\right)}{3 c e \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}$$

$$+\frac{b \sqrt{c^2 x^2-1}\, d^2 \ln\left(c x+\sqrt{c^2 x^2-1}\right)}{c^2 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}+\frac{b e^2 x^2}{6 c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{b e^2}{6 c^3 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}+\frac{b e x d}{c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{b e d}{c^3 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}$$

$$+\frac{b e^2 \sqrt{c^2 x^2-1}\, \ln\left(c x+\sqrt{c^2 x^2-1}\right)}{6 c^4 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{a+b \operatorname{arccsc}(c x)}{(e x+d)^2}\, dx$$

Optimal(type 3, 98 leaves, 7 steps):

$$\frac{b \operatorname{arccsc}(c x)}{d e}+\frac{-a-b \operatorname{arccsc}(c x)}{e (e x+d)}+\frac{b \operatorname{arctanh}\left(\frac{c^2 d+\frac{e}{x}}{c \sqrt{c^2 d^2-e^2}\, \sqrt{1-\frac{1}{c^2 x^2}}}\right)}{d \sqrt{c^2 d^2-e^2}}$$

Result(type 3, 213 leaves):

$$-\frac{c a}{(c e x+c d) e}-\frac{c b \operatorname{arccsc}(c x)}{(c e x+c d) e}+\frac{b \sqrt{c^2 x^2-1}\, \arctan\left(\frac{1}{\sqrt{c^2 x^2-1}}\right)}{c e \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x d}-\frac{b \sqrt{c^2 x^2-1}\, \ln\left(\frac{2 \left(\sqrt{c^2 x^2-1}\, \sqrt{\frac{c^2 d^2-e^2}{e^2}}\, e-c^2 d x-e\right)}{c e x+c d}\right)}{c e \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x d \sqrt{\frac{c^2 d^2-e^2}{e^2}}}$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int (e x+d)^{3/2} (a+b \operatorname{arccsc}(c x))\, dx$$

Optimal(type 4, 335 leaves, 22 steps):

$$\frac{2 (e x+d)^{5/2} (a+b \operatorname{arccsc}(c x))}{5 e}-\frac{4 b e (-c^2 x^2+1) \sqrt{e x+d}}{15 c^3 x \sqrt{1-\frac{1}{c^2 x^2}}}-\frac{28 b d \operatorname{EllipticE}\left(\frac{\sqrt{-c x+1} \sqrt{2}}{2}, \sqrt{2} \sqrt{\frac{e}{c d+e}}\right) \sqrt{e x+d} \sqrt{-c^2 x^2+1}}{15 c^2 x \sqrt{1-\frac{1}{c^2 x^2}} \sqrt{\frac{c (e x+d)}{c d+e}}}$$

$$-\frac{4 b (2 c^2 d^2+e^2) \operatorname{EllipticF}\left(\frac{\sqrt{-c x+1} \sqrt{2}}{2}, \sqrt{2} \sqrt{\frac{e}{c d+e}}\right) \sqrt{\frac{c (e x+d)}{c d+e}} \sqrt{-c^2 x^2+1}}{15 c^4 x \sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e x+d}}$$

$$-\frac{4 b d^3 \operatorname{EllipticPi}\left(\frac{\sqrt{-c x+1} \sqrt{2}}{2}, 2, \sqrt{2} \sqrt{\frac{e}{c d+e}}\right) \sqrt{\frac{c (e x+d)}{c d+e}} \sqrt{-c^2 x^2+1}}{5 c e x \sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e x+d}}$$

Result(type 4, 809 leaves):

$$\frac{1}{e}\left(2\left(\frac{(e x+d)^{5/2} a}{5}+b\left(\frac{(e x+d)^{5/2} \operatorname{arccsc}(c x)}{5}+\frac{1}{15 c^3 \sqrt{\frac{c}{c d-e}}\, x \sqrt{\frac{(e x+d)^2 c^2-2 (e x+d) c^2 d+c^2 d^2-e^2}{c^2 x^2 e^2}}}\left(2\left(\sqrt{\frac{c}{c d-e}}\, (e x+d)^{5/}\right.\right.\right.\right.\right.$$

$$^{2} c^2+9 d^2 \sqrt{-\frac{c (e x+d)-c d+e}{c d-e}} \sqrt{-\frac{c (e x+d)-c d-e}{c d+e}} \operatorname{EllipticF}\left(\sqrt{e x+d} \sqrt{\frac{c}{c d-e}}, \sqrt{\frac{c d-e}{c d+e}}\right) c^2$$

$$-7 \sqrt{-\frac{c (e x+d)-c d+e}{c d-e}} \sqrt{-\frac{c (e x+d)-c d-e}{c d+e}} \operatorname{EllipticE}\left(\sqrt{e x+d} \sqrt{\frac{c}{c d-e}}, \sqrt{\frac{c d-e}{c d+e}}\right) c^2 d^2$$

$$-3 d^2 \sqrt{-\frac{c (e x+d)-c d+e}{c d-e}} \sqrt{-\frac{c (e x+d)-c d-e}{c d+e}} \operatorname{EllipticPi}\left(\sqrt{e x+d} \sqrt{\frac{c}{c d-e}}, \frac{c d-e}{c d}, \frac{\sqrt{\frac{c}{c d+e}}}{\sqrt{\frac{c}{c d-e}}}\right) c^2-2 \sqrt{\frac{c}{c d-e}}\, (e x+d)^{3/2} c^2 d$$

$$+7 \sqrt{-\frac{c (e x+d)-c d+e}{c d-e}} \sqrt{-\frac{c (e x+d)-c d-e}{c d+e}} \operatorname{EllipticF}\left(\sqrt{e x+d} \sqrt{\frac{c}{c d-e}}, \sqrt{\frac{c d-e}{c d+e}}\right) c d e$$

$$-7\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\mathrm{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cde+\sqrt{\frac{c}{cd-e}}\sqrt{ex+d}\,c^2d^2$$

$$+\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)e^2-\sqrt{\frac{c}{cd-e}}\sqrt{ex+d}\,e^2\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)$$

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Problem 23: Result more than twice size of optimal antiderivative.
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$$\int\frac{x^2(a+b\,\mathrm{arccsc}(cx))}{(ex+d)^{5/2}}\,dx$$

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Optimal(type 4, 401 leaves, 25 steps):
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$$-\frac{2d^2(a+b\,\mathrm{arccsc}(cx))}{3e^3(ex+d)^{3/2}}+\frac{4d(a+b\,\mathrm{arccsc}(cx))}{e^3\sqrt{ex+d}}+\frac{4bd(-c^2x^2+1)}{3ce(c^2d^2-e^2)x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{ex+d}}+\frac{2(a+b\,\mathrm{arccsc}(cx))\sqrt{ex+d}}{e^3}$$

$$-\frac{4bd\,\mathrm{EllipticE}\left(\frac{\sqrt{-cx+1}\sqrt{2}}{2},\sqrt{2}\sqrt{\frac{e}{cd+e}}\right)\sqrt{ex+d}\sqrt{-c^2x^2+1}}{3e^2(c^2d^2-e^2)x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(ex+d)}{cd+e}}}$$

$$-\frac{4b\,\mathrm{EllipticF}\left(\frac{\sqrt{-cx+1}\sqrt{2}}{2},\sqrt{2}\sqrt{\frac{e}{cd+e}}\right)\sqrt{\frac{c(ex+d)}{cd+e}}\sqrt{-c^2x^2+1}}{c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{ex+d}}$$

$$-\frac{32bd\,\mathrm{EllipticPi}\left(\frac{\sqrt{-cx+1}\sqrt{2}}{2},2,\sqrt{2}\sqrt{\frac{e}{cd+e}}\right)\sqrt{\frac{c(ex+d)}{cd+e}}\sqrt{-c^2x^2+1}}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{ex+d}}$$

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Result(type 4, 1039 leaves):
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$$\frac{1}{e^3}\Bigg(2\Bigg(a\left(\sqrt{ex+d}-\frac{d^2}{3(ex+d)^{3/2}}+\frac{2d}{\sqrt{ex+d}}\right)+b\Bigg(\mathrm{arccsc}(cx)\sqrt{ex+d}-\frac{\mathrm{arccsc}(cx)\,d^2}{3(ex+d)^{3/2}}+\frac{2\,\mathrm{arccsc}(cx)\,d}{\sqrt{ex+d}}$$

$$+\left(2\left(4\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)\sqrt{ex+d}\,c^2d^2-\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(e\ldots}{}}\right.\right.$$

$$\left.\sqrt{\frac{cd-e}{cd+e}}\right)\sqrt{ex+d}\,c^2d^2-8\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\mathrm{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\right.$$

$$\left.\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)\sqrt{ex+d}\,c^2d^2-\sqrt{\frac{c}{cd-e}}\,(ex+d)^2c^2d+\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\right.$$

$$\left.\sqrt{\frac{cd-e}{cd+e}}\right)\sqrt{ex+d}\,cde-\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\mathrm{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)\sqrt{ex+d}\,cde$$

$$+2\sqrt{\frac{c}{cd-e}}\,(ex+d)\,c^2d^2-3\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)\sqrt{ex+d}\,e^2$$

$$+8\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\mathrm{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)\sqrt{ex+d}\,e^2-\sqrt{\frac{c}{cd-e}}\,c^2d^3$$

$$\left.\left.+\sqrt{\frac{c}{cd-e}}\,de^2\right)\right)\Bigg/\left(3c(cd-e)\sqrt{\frac{c}{cd-e}}\sqrt{ex+d}\,(cd+e)\,x\sqrt{\frac{(ex+d)^2c^2-2(ex+d)c^2d+c^2d^2-e^2}{c^2x^2e^2}}\right)$$

Problem 28: Result is not expressed in closed-form.

$$\int\frac{a+b\,\mathrm{arccsc}(cx)}{ex^2+d}\,dx$$

Optimal(type 4, 517 leaves, 19 steps):

$$-\frac{(a+b\operatorname{arccsc}(cx))\ln\left(1-\frac{\mathrm{I}c\left(\frac{\mathrm{I}}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\operatorname{arccsc}(cx))\ln\left(1+\frac{\mathrm{I}c\left(\frac{\mathrm{I}}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

$$-\frac{(a+b\operatorname{arccsc}(cx))\ln\left(1-\frac{\mathrm{I}c\left(\frac{\mathrm{I}}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\operatorname{arccsc}(cx))\ln\left(1+\frac{\mathrm{I}c\left(\frac{\mathrm{I}}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

$$-\frac{\mathrm{I}b\operatorname{polylog}\left(2,\frac{-\mathrm{I}c\left(\frac{\mathrm{I}}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\mathrm{I}b\operatorname{polylog}\left(2,\frac{\mathrm{I}c\left(\frac{\mathrm{I}}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

$$-\frac{\mathrm{I}b\operatorname{polylog}\left(2,\frac{-\mathrm{I}c\left(\frac{\mathrm{I}}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\mathrm{I}b\operatorname{polylog}\left(2,\frac{\mathrm{I}c\left(\frac{\mathrm{I}}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

Result(type 7, 271 leaves):

$$\frac{a\arctan\left(\frac{xe}{\sqrt{ed}}\right)}{\sqrt{ed}}-\frac{cb\left(\sum_{\_R1=RootOf(c^2d\_Z^4+(-2c^2d-4e)\_Z^2+c^2d)}\frac{\mathrm{I}\operatorname{arccsc}(cx)\ln\left(\frac{\_R1-\frac{\mathrm{I}}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{\mathrm{I}}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)}{\_R1\left(\_R1^2c^2d-c^2d-2e\right)}\right)}{2}$$

$$-\frac{cb\left(\sum_{\_R1=RootOf(c^2d\_Z^4+(-2c^2d-4e)\_Z^2+c^2d)}\frac{\_R1\left(\mathrm{I}\operatorname{arccsc}(cx)\ln\left(\frac{\_R1-\frac{\mathrm{I}}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{\mathrm{I}}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)\right)}{\_R1^2c^2d-c^2d-2e}\right)}{2}$$

Problem 29: Result is not expressed in closed-form.

$$\int\frac{a+b\operatorname{arccsc}(cx)}{x\left(ex^2+d\right)}\,dx$$

Optimal(type 4, 496 leaves, 19 steps):

$$\frac{I\,(a+b\,\mathrm{arccsc}(c\,x))^2}{2\,b\,d}-\frac{(a+b\,\mathrm{arccsc}(c\,x))\ln\left(1-\frac{I\,c\left(\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right)}{2\,d}-\frac{(a+b\,\mathrm{arccsc}(c\,x))\ln\left(1+\frac{I\,c\left(\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right)}{2\,d}$$

$$-\frac{(a+b\,\mathrm{arccsc}(c\,x))\ln\left(1-\frac{I\,c\left(\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right)}{2\,d}-\frac{(a+b\,\mathrm{arccsc}(c\,x))\ln\left(1+\frac{I\,c\left(\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right)}{2\,d}$$

$$+\frac{I\,b\,\mathrm{polylog}\left(2,\frac{-I\,c\left(\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right)}{2\,d}+\frac{I\,b\,\mathrm{polylog}\left(2,\frac{I\,c\left(\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right)}{2\,d}$$

$$+\frac{I\,b\,\mathrm{polylog}\left(2,\frac{-I\,c\left(\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right)}{2\,d}+\frac{I\,b\,\mathrm{polylog}\left(2,\frac{I\,c\left(\frac{I}{c\,x}+\sqrt{1-\frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right)}{2\,d}$$

Result(type 7, 447 leaves):

$$\frac{a\ln(c\,x)}{d}-\frac{a\ln\left(c^2\,e\,x^2+c^2\,d\right)}{2\,d}+\frac{I\,b\,\mathrm{arccsc}(c\,x)^2}{2\,d}$$

$$+\frac{I\,b\,c^2\left(\sum_{\_R1=RootOf\left(c^2\,d\,\_Z^4+(-2\,c^2\,d-4\,e)\,\_Z^2+c^2\,d\right)}\frac{I\,\mathrm{arccsc}(c\,x)\ln\left(\frac{\_R1-\frac{I}{c\,x}-\sqrt{1-\frac{1}{c^2\,x^2}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{I}{c\,x}-\sqrt{1-\frac{1}{c^2\,x^2}}}{\_R1}\right)}{\_R1^2\,c^2\,d-c^2\,d-2\,e}\right)}{2}$$

$$+\frac{1}{2\,d}\Bigg(I\,b\,\Bigg($$

$$\sum_{\_R1=RootOf\left(c^2\,d\,\_Z^4+(-2\,c^2\,d-4\,e)\,\_Z^2+c^2\,d\right)}$$

$$\frac{\left(\_R1^2 c^2 d - 2 c^2 d - 4 e\right) \left(\mathrm{I} \operatorname{arccsc}(c x) \ln\left(\frac{\_R1 - \frac{\mathrm{I}}{c x} - \sqrt{1 - \frac{1}{c^2 x^2}}}{\_R1}\right) + \operatorname{dilog}\left(\frac{\_R1 - \frac{\mathrm{I}}{c x} - \sqrt{1 - \frac{1}{c^2 x^2}}}{\_R1}\right)\right)}{\_R1^2 c^2 d - c^2 d - 2 e}\Bigg)\Bigg)$$

$$+ \frac{\mathrm{I} b e \left(\sum_{\_R1 = RootOf(c^2 d \_Z^4 + (-2 c^2 d - 4 e) \_Z^2 + c^2 d)} \frac{\mathrm{I} \operatorname{arccsc}(c x) \ln\left(\frac{\_R1 - \frac{\mathrm{I}}{c x} - \sqrt{1 - \frac{1}{c^2 x^2}}}{\_R1}\right) + \operatorname{dilog}\left(\frac{\_R1 - \frac{\mathrm{I}}{c x} - \sqrt{1 - \frac{1}{c^2 x^2}}}{\_R1}\right)}{\_R1^2 c^2 d - c^2 d - 2 e}\right)}{d}$$

Problem 30: Result is not expressed in closed-form.

$$\int \frac{a + b \operatorname{arccsc}(c x)}{x^2 \left(e x^2 + d\right)} \, dx$$

Optimal(type 4, 558 leaves, 24 steps):

$$-\frac{a}{d x} - \frac{b \operatorname{arccsc}(c x)}{d x} - \frac{(a + b \operatorname{arccsc}(c x)) \ln\left(1 - \frac{\mathrm{I} c \left(\frac{\mathrm{I}}{c x} + \sqrt{1 - \frac{1}{c^2 x^2}}\right) \sqrt{-d}}{\sqrt{e} - \sqrt{c^2 d + e}}\right) \sqrt{e}}{2 (-d)^{3/2}}$$

$$+ \frac{(a + b \operatorname{arccsc}(c x)) \ln\left(1 + \frac{\mathrm{I} c \left(\frac{\mathrm{I}}{c x} + \sqrt{1 - \frac{1}{c^2 x^2}}\right) \sqrt{-d}}{\sqrt{e} - \sqrt{c^2 d + e}}\right) \sqrt{e}}{2 (-d)^{3/2}} - \frac{(a + b \operatorname{arccsc}(c x)) \ln\left(1 - \frac{\mathrm{I} c \left(\frac{\mathrm{I}}{c x} + \sqrt{1 - \frac{1}{c^2 x^2}}\right) \sqrt{-d}}{\sqrt{e} + \sqrt{c^2 d + e}}\right) \sqrt{e}}{2 (-d)^{3/2}}$$

$$+ \frac{(a + b \operatorname{arccsc}(c x)) \ln\left(1 + \frac{\mathrm{I} c \left(\frac{\mathrm{I}}{c x} + \sqrt{1 - \frac{1}{c^2 x^2}}\right) \sqrt{-d}}{\sqrt{e} + \sqrt{c^2 d + e}}\right) \sqrt{e}}{2 (-d)^{3/2}} - \frac{\mathrm{I} b \operatorname{polylog}\left(2, \frac{-\mathrm{I} c \left(\frac{\mathrm{I}}{c x} + \sqrt{1 - \frac{1}{c^2 x^2}}\right) \sqrt{-d}}{\sqrt{e} - \sqrt{c^2 d + e}}\right) \sqrt{e}}{2 (-d)^{3/2}}$$

$$+ \frac{\mathrm{I} b \operatorname{polylog}\left(2, \frac{\mathrm{I} c \left(\frac{\mathrm{I}}{c x} + \sqrt{1 - \frac{1}{c^2 x^2}}\right) \sqrt{-d}}{\sqrt{e} - \sqrt{c^2 d + e}}\right) \sqrt{e}}{2 (-d)^{3/2}} - \frac{\mathrm{I} b \operatorname{polylog}\left(2, \frac{-\mathrm{I} c \left(\frac{\mathrm{I}}{c x} + \sqrt{1 - \frac{1}{c^2 x^2}}\right) \sqrt{-d}}{\sqrt{e} + \sqrt{c^2 d + e}}\right) \sqrt{e}}{2 (-d)^{3/2}}$$

$$+ \frac{\mathrm{I} b \operatorname{polylog}\left(2, \frac{\mathrm{I} c \left(\frac{\mathrm{I}}{c x} + \sqrt{1 - \frac{1}{c^2 x^2}}\right) \sqrt{-d}}{\sqrt{e} + \sqrt{c^2 d + e}}\right) \sqrt{e}}{2 (-d)^{3/2}} - \frac{b c \sqrt{1 - \frac{1}{c^2 x^2}}}{d}$$

Result(type 7, 331 leaves):

$$-\frac{a\,e\arctan\left(\frac{xe}{\sqrt{ed}}\right)}{d\sqrt{ed}}-\frac{a}{dx}-\frac{cb\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arccsc}(cx)}{dx}$$
$$+\frac{cbe\left(\sum_{\_R1=RootOf(c^2d\_Z^4+(-2c^2d-4e)\_Z^2+c^2d)}\frac{\mathrm{I}\,\mathrm{arccsc}(cx)\ln\left(\frac{\_R1-\frac{\mathrm{I}}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{\mathrm{I}}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)}{\_R1\,(\_R1^2c^2d-c^2d-2e)}\right)}{2d}$$
$$+\frac{cbe\left(\sum_{\_R1=RootOf(c^2d\_Z^4+(-2c^2d-4e)\_Z^2+c^2d)}\frac{\_R1\left(\mathrm{I}\,\mathrm{arccsc}(cx)\ln\left(\frac{\_R1-\frac{\mathrm{I}}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{\mathrm{I}}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)\right)}{\_R1^2c^2d-c^2d-2e}\right)}{2d}$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int\frac{x\,(a+b\,\mathrm{arccsc}(cx))}{(ex^2+d)^3}\,dx$$

Optimal(type 3, 168 leaves, 8 steps):

$$\frac{-a-b\,\mathrm{arccsc}(cx)}{4e\,(ex^2+d)^2}-\frac{bcx\arctan\left(\sqrt{c^2x^2-1}\right)}{4d^2e\sqrt{c^2x^2}}+\frac{bc\,(3c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8d^2\,(c^2d+e)^{3/2}\sqrt{e}\sqrt{c^2x^2}}+\frac{bcx\sqrt{c^2x^2-1}}{8d\,(c^2d+e)\,(ex^2+d)\sqrt{c^2x^2}}$$

Result(type 3, 1839 leaves):

$$-\frac{c^4a}{4e\,(c^2ex^2+c^2d)^2}-\frac{c^4b\,\mathrm{arccsc}(cx)}{4e\,(c^2ex^2+c^2d)^2}-\frac{c^3b\sqrt{c^2x^2-1}\,x\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)e}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,d\,(c^2d+e)\left(-cxe+\sqrt{-edc^2}\right)\left(cxe+\sqrt{-edc^2}\right)}$$
$$-\frac{c^3b\sqrt{c^2x^2-1}\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x\,(c^2d+e)\left(-cxe+\sqrt{-edc^2}\right)\left(cxe+\sqrt{-edc^2}\right)}$$

$$+\frac{3 c^3 b \sqrt{c^2 x^2-1}\, x \ln\left(-\frac{2\left(\sqrt{-\frac{c^2 d+e}{e}}\,\sqrt{c^2 x^2-1}\, e+\sqrt{-e d c^2}\, c x-e\right)}{-c x e+\sqrt{-e d c^2}}\right) e}{16 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, d \sqrt{-\frac{c^2 d+e}{e}}\, (c^2 d+e)\left(-c x e+\sqrt{-e d c^2}\right)\left(c x e+\sqrt{-e d c^2}\right)}$$

$$+\frac{3 c^3 b \sqrt{c^2 x^2-1}\, \ln\left(-\frac{2\left(\sqrt{-\frac{c^2 d+e}{e}}\,\sqrt{c^2 x^2-1}\, e+\sqrt{-e d c^2}\, c x-e\right)}{-c x e+\sqrt{-e d c^2}}\right)}{16 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x \sqrt{-\frac{c^2 d+e}{e}}\, (c^2 d+e)\left(-c x e+\sqrt{-e d c^2}\right)\left(c x e+\sqrt{-e d c^2}\right)}$$

$$+\frac{3 c^3 b \sqrt{c^2 x^2-1}\, x \ln\left(\frac{2\left(\sqrt{-\frac{c^2 d+e}{e}}\,\sqrt{c^2 x^2-1}\, e-\sqrt{-e d c^2}\, c x-e\right)}{c x e+\sqrt{-e d c^2}}\right) e}{16 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, d \sqrt{-\frac{c^2 d+e}{e}}\, (c^2 d+e)\left(-c x e+\sqrt{-e d c^2}\right)\left(c x e+\sqrt{-e d c^2}\right)}$$

$$+\frac{3 c^3 b \sqrt{c^2 x^2-1}\, \ln\left(\frac{2\left(\sqrt{-\frac{c^2 d+e}{e}}\,\sqrt{c^2 x^2-1}\, e-\sqrt{-e d c^2}\, c x-e\right)}{c x e+\sqrt{-e d c^2}}\right)}{16 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x \sqrt{-\frac{c^2 d+e}{e}}\, (c^2 d+e)\left(-c x e+\sqrt{-e d c^2}\right)\left(c x e+\sqrt{-e d c^2}\right)}$$

$$-\frac{c b \sqrt{c^2 x^2-1}\, x \arctan\left(\frac{1}{\sqrt{c^2 x^2-1}}\right) e^2}{4 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, d^2 (c^2 d+e)\left(-c x e+\sqrt{-e d c^2}\right)\left(c x e+\sqrt{-e d c^2}\right)} - \frac{c b \sqrt{c^2 x^2-1}\, \arctan\left(\frac{1}{\sqrt{c^2 x^2-1}}\right) e}{4 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x d (c^2 d+e)\left(-c x e+\sqrt{-e d c^2}\right)\left(c x e+\sqrt{-e d c^2}\right)}$$

$$-\frac{c^3 b x e}{8 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, d (c^2 d+e)\left(-c x e+\sqrt{-e d c^2}\right)\left(c x e+\sqrt{-e d c^2}\right)} + \frac{c b e}{8 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x d (c^2 d+e)\left(-c x e+\sqrt{-e d c^2}\right)\left(c x e+\sqrt{-e d c^2}\right)}$$

$$+\frac{c b \sqrt{c^2 x^2-1}\, x \ln\left(-\frac{2\left(\sqrt{-\frac{c^2 d+e}{e}}\,\sqrt{c^2 x^2-1}\, e+\sqrt{-e d c^2}\, c x-e\right)}{-c x e+\sqrt{-e d c^2}}\right) e^2}{8 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, d^2 \sqrt{-\frac{c^2 d+e}{e}}\, (c^2 d+e)\left(-c x e+\sqrt{-e d c^2}\right)\left(c x e+\sqrt{-e d c^2}\right)}$$

$$+\frac{c b \sqrt{c^2 x^2-1}\, \ln\left(-\frac{2\left(\sqrt{-\frac{c^2 d+e}{e}}\, \sqrt{c^2 x^2-1}\, e+\sqrt{-e d c^2}\, c x-e\right)}{-c x e+\sqrt{-e d c^2}}\right) e}{8 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x d \sqrt{-\frac{c^2 d+e}{e}}\, (c^2 d+e) \left(-c x e+\sqrt{-e d c^2}\right) \left(c x e+\sqrt{-e d c^2}\right)}$$

$$+\frac{c b \sqrt{c^2 x^2-1}\, x \ln\left(\frac{2\left(\sqrt{-\frac{c^2 d+e}{e}}\, \sqrt{c^2 x^2-1}\, e-\sqrt{-e d c^2}\, c x-e\right)}{c x e+\sqrt{-e d c^2}}\right) e^2}{8 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, d^2 \sqrt{-\frac{c^2 d+e}{e}}\, (c^2 d+e) \left(-c x e+\sqrt{-e d c^2}\right) \left(c x e+\sqrt{-e d c^2}\right)}$$

$$+\frac{c b \sqrt{c^2 x^2-1}\, \ln\left(\frac{2\left(\sqrt{-\frac{c^2 d+e}{e}}\, \sqrt{c^2 x^2-1}\, e-\sqrt{-e d c^2}\, c x-e\right)}{c x e+\sqrt{-e d c^2}}\right) e}{8 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x d \sqrt{-\frac{c^2 d+e}{e}}\, (c^2 d+e) \left(-c x e+\sqrt{-e d c^2}\right) \left(c x e+\sqrt{-e d c^2}\right)}$$

Problem 32: Result is not expressed in closed-form.

$$\int \frac{a+b\,\mathrm{arccsc}(c x)}{(e x^2+d)^3}\, dx$$

Optimal(type 4, 1002 leaves, 81 steps):

$$-\frac{b e \,\mathrm{arctanh}\left(\frac{c^2 d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d+e}\,\sqrt{1-\frac{1}{c^2 x^2}}}\right)}{16 d^{5/2} (c^2 d+e)^{3/2}}-\frac{b e \,\mathrm{arctanh}\left(\frac{c^2 d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d+e}\,\sqrt{1-\frac{1}{c^2 x^2}}}\right)}{16 d^{5/2} (c^2 d+e)^{3/2}}$$

$$-\frac{3 (a+b\,\mathrm{arccsc}(c x)) \ln\left(1-\frac{I c\left(\frac{I}{c x}+\sqrt{1-\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{16 (-d)^{5/2} \sqrt{e}}+\frac{3 (a+b\,\mathrm{arccsc}(c x)) \ln\left(1+\frac{I c\left(\frac{I}{c x}+\sqrt{1-\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{16 (-d)^{5/2} \sqrt{e}}$$

$$-\frac{3\left(a+b\,\mathrm{arccsc}(cx)\right)\ln\left(1-\frac{I c\left(\frac{I}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\,(-d)^{5/2}\sqrt{e}}+\frac{3\left(a+b\,\mathrm{arccsc}(cx)\right)\ln\left(1+\frac{I c\left(\frac{I}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\,(-d)^{5/2}\sqrt{e}}$$

$$-\frac{3 I b\,\mathrm{polylog}\left(2,\frac{-I c\left(\frac{I}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\,(-d)^{5/2}\sqrt{e}}-\frac{3 I b\,\mathrm{polylog}\left(2,\frac{-I c\left(\frac{I}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\,(-d)^{5/2}\sqrt{e}}$$

$$+\frac{3 I b\,\mathrm{polylog}\left(2,\frac{I c\left(\frac{I}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\,(-d)^{5/2}\sqrt{e}}+\frac{3 I b\,\mathrm{polylog}\left(2,\frac{I c\left(\frac{I}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\,(-d)^{5/2}\sqrt{e}}+\frac{\left(a+b\,\mathrm{arccsc}(cx)\right)\sqrt{e}}{16\,(-d)^{3/2}\left(-\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)^2}$$

$$-\frac{5\left(a+b\,\mathrm{arccsc}(cx)\right)}{16\,d^2\left(-\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}-\frac{\left(a+b\,\mathrm{arccsc}(cx)\right)\sqrt{e}}{16\,(-d)^{3/2}\left(\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)^2}+\frac{5\left(a+b\,\mathrm{arccsc}(cx)\right)}{16\,d^2\left(\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}+\frac{5 b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d+e}\,\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\,d^{5/2}\sqrt{c^2d+e}}$$

$$+\frac{5 b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d+e}\,\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\,d^{5/2}\sqrt{c^2d+e}}-\frac{b c\sqrt{e}\,\sqrt{1-\frac{1}{c^2x^2}}}{16\,(-d)^{3/2}\left(c^2d+e\right)\left(-\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}-\frac{b c\sqrt{e}\,\sqrt{1-\frac{1}{c^2x^2}}}{16\,(-d)^{3/2}\left(c^2d+e\right)\left(\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}$$

Result(type ?, 3165 leaves): Display of huge result suppressed!

Problem 33: Unable to integrate problem.

$$\int x^5\left(a+b\,\mathrm{arccsc}(cx)\right)\sqrt{ex^2+d}\,dx$$

Optimal(type 3, 343 leaves, 12 steps):

$$\frac{d^2\left(ex^2+d\right)^{3/2}\left(a+b\,\mathrm{arccsc}(cx)\right)}{3\,e^3}-\frac{2\,d\left(ex^2+d\right)^{5/2}\left(a+b\,\mathrm{arccsc}(cx)\right)}{5\,e^3}+\frac{\left(ex^2+d\right)^{7/2}\left(a+b\,\mathrm{arccsc}(cx)\right)}{7\,e^3}$$

$$-\frac{8 b c d^{7/2} x \arctan\left(\frac{\sqrt{e x^2+d}}{\sqrt{d}\sqrt{c^2 x^2-1}}\right)}{105 e^3 \sqrt{c^2 x^2}}+\frac{b \left(105 d^3 c^6-35 c^4 d^2 e+63 c^2 d e^2+75 e^3\right) x \operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2 x^2-1}}{c \sqrt{e x^2+d}}\right)}{1680 c^6 e^{5/2} \sqrt{c^2 x^2}}$$

$$-\frac{b \left(29 c^2 d-25 e\right) x \left(e x^2+d\right)^{3/2} \sqrt{c^2 x^2-1}}{840 c^3 e^2 \sqrt{c^2 x^2}}+\frac{b x \left(e x^2+d\right)^{5/2} \sqrt{c^2 x^2-1}}{42 c e^2 \sqrt{c^2 x^2}}-\frac{b \left(23 c^4 d^2+12 e d c^2-75 e^2\right) x \sqrt{c^2 x^2-1}\sqrt{e x^2+d}}{1680 c^5 e^2 \sqrt{c^2 x^2}}$$

Result(type 8, 23 leaves):

$$\int x^5 \left(a+b \operatorname{arccsc}(c x)\right) \sqrt{e x^2+d}\, dx$$

Problem 34: Unable to integrate problem.

$$\int x^3 \left(a+b \operatorname{arccsc}(c x)\right) \sqrt{e x^2+d}\, dx$$

Optimal(type 3, 246 leaves, 11 steps):

$$-\frac{d \left(e x^2+d\right)^{3/2} \left(a+b \operatorname{arccsc}(c x)\right)}{3 e^2}+\frac{\left(e x^2+d\right)^{5/2} \left(a+b \operatorname{arccsc}(c x)\right)}{5 e^2}+\frac{2 b c d^{5/2} x \arctan\left(\frac{\sqrt{e x^2+d}}{\sqrt{d}\sqrt{c^2 x^2-1}}\right)}{15 e^2 \sqrt{c^2 x^2}}$$

$$-\frac{b \left(15 c^4 d^2-10 e d c^2-9 e^2\right) x \operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2 x^2-1}}{c \sqrt{e x^2+d}}\right)}{120 c^4 e^{3/2} \sqrt{c^2 x^2}}+\frac{b x \left(e x^2+d\right)^{3/2} \sqrt{c^2 x^2-1}}{20 c e \sqrt{c^2 x^2}}+\frac{b \left(c^2 d+9 e\right) x \sqrt{c^2 x^2-1}\sqrt{e x^2+d}}{120 c^3 e \sqrt{c^2 x^2}}$$

Result(type 8, 23 leaves):

$$\int x^3 \left(a+b \operatorname{arccsc}(c x)\right) \sqrt{e x^2+d}\, dx$$

Problem 35: Unable to integrate problem.

$$\int x \left(a+b \operatorname{arccsc}(c x)\right) \sqrt{e x^2+d}\, dx$$

Optimal(type 3, 159 leaves, 9 steps):

$$\frac{\left(e x^2+d\right)^{3/2} \left(a+b \operatorname{arccsc}(c x)\right)}{3 e}-\frac{b c d^{3/2} x \arctan\left(\frac{\sqrt{e x^2+d}}{\sqrt{d}\sqrt{c^2 x^2-1}}\right)}{3 e \sqrt{c^2 x^2}}+\frac{b \left(3 c^2 d+e\right) x \operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2 x^2-1}}{c \sqrt{e x^2+d}}\right)}{6 c^2 \sqrt{e}\sqrt{c^2 x^2}}+\frac{b x \sqrt{c^2 x^2-1}\sqrt{e x^2+d}}{6 c \sqrt{c^2 x^2}}$$

Result(type 8, 21 leaves):

$$\int x \left(a+b \operatorname{arccsc}(c x)\right) \sqrt{e x^2+d}\, dx$$

Problem 38: Unable to integrate problem.

$$\int \frac{\left(a+b\,\mathrm{arccsc}(c\,x)\right)\sqrt{e\,x^2+d}}{x^4}\,dx$$

Optimal(type 4, 286 leaves, 11 steps):

$$-\frac{\left(e\,x^2+d\right)^{3/2}\left(a+b\,\mathrm{arccsc}(c\,x)\right)}{3\,d\,x^3}-\frac{2\,b\,c\left(c^2\,d+2\,e\right)\sqrt{c^2\,x^2-1}\,\sqrt{e\,x^2+d}}{9\,d\sqrt{c^2\,x^2}}-\frac{b\,c\sqrt{c^2\,x^2-1}\,\sqrt{e\,x^2+d}}{9\,x^2\sqrt{c^2\,x^2}}$$
$$+\frac{2\,b\,c^2\left(c^2\,d+2\,e\right)x\,\mathrm{EllipticE}\left(c\,x,\sqrt{-\frac{e}{c^2\,d}}\right)\sqrt{-c^2\,x^2+1}\,\sqrt{e\,x^2+d}}{9\,d\sqrt{c^2\,x^2}\,\sqrt{c^2\,x^2-1}\,\sqrt{1+\frac{e\,x^2}{d}}}$$
$$-\frac{b\left(c^2\,d+e\right)\left(2\,c^2\,d+3\,e\right)x\,\mathrm{EllipticF}\left(c\,x,\sqrt{-\frac{e}{c^2\,d}}\right)\sqrt{-c^2\,x^2+1}\,\sqrt{1+\frac{e\,x^2}{d}}}{9\,d\sqrt{c^2\,x^2}\,\sqrt{c^2\,x^2-1}\,\sqrt{e\,x^2+d}}$$

Result(type 8, 23 leaves):

$$\int \frac{\left(a+b\,\mathrm{arccsc}(c\,x)\right)\sqrt{e\,x^2+d}}{x^4}\,dx$$

Problem 42: Unable to integrate problem.

$$\int \frac{x^5\left(a+b\,\mathrm{arccsc}(c\,x)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

Optimal(type 3, 212 leaves, 10 steps):

$$\frac{\left(e\,x^2+d\right)^{3/2}\left(a+b\,\mathrm{arccsc}(c\,x)\right)}{3\,e^3}+\frac{8\,b\,c\,d^{3/2}\,x\arctan\left(\frac{\sqrt{e\,x^2+d}}{\sqrt{d}\,\sqrt{c^2\,x^2-1}}\right)}{3\,e^3\sqrt{c^2\,x^2}}-\frac{b\left(9\,c^2\,d-e\right)x\,\mathrm{arctanh}\left(\frac{\sqrt{e}\,\sqrt{c^2\,x^2-1}}{c\sqrt{e\,x^2+d}}\right)}{6\,c^2\,e^{5/2}\sqrt{c^2\,x^2}}-\frac{d^2\left(a+b\,\mathrm{arccsc}(c\,x)\right)}{e^3\sqrt{e\,x^2+d}}$$
$$-\frac{2\,d\left(a+b\,\mathrm{arccsc}(c\,x)\right)\sqrt{e\,x^2+d}}{e^3}+\frac{b\,x\sqrt{c^2\,x^2-1}\,\sqrt{e\,x^2+d}}{6\,c\,e^2\sqrt{c^2\,x^2}}$$

Result(type 8, 23 leaves):

$$\int \frac{x^5\left(a+b\,\mathrm{arccsc}(c\,x)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

Problem 43: Unable to integrate problem.

$$\int \frac{x^3 (a + b \operatorname{arccsc}(c x))}{(e x^2 + d)^{3/2}} \, dx$$

Optimal(type 3, 132 leaves, 9 steps):

$$\frac{b x \operatorname{arctanh}\left(\frac{\sqrt{e} \sqrt{c^2 x^2 - 1}}{c \sqrt{e x^2 + d}}\right)}{e^{3/2} \sqrt{c^2 x^2}} - \frac{2 b c x \arctan\left(\frac{\sqrt{e x^2 + d}}{\sqrt{d} \sqrt{c^2 x^2 - 1}}\right) \sqrt{d}}{e^2 \sqrt{c^2 x^2}} + \frac{d (a + b \operatorname{arccsc}(c x))}{e^2 \sqrt{e x^2 + d}} + \frac{(a + b \operatorname{arccsc}(c x)) \sqrt{e x^2 + d}}{e^2}$$

Result(type 8, 23 leaves):

$$\int \frac{x^3 (a + b \operatorname{arccsc}(c x))}{(e x^2 + d)^{3/2}} \, dx$$

Problem 45: Unable to integrate problem.

$$\int \frac{a + b \operatorname{arccsc}(c x)}{x^2 (e x^2 + d)^{3/2}} \, dx$$

Optimal(type 4, 249 leaves, 10 steps):

$$\frac{-a - b \operatorname{arccsc}(c x)}{d x \sqrt{e x^2 + d}} - \frac{2 e x (a + b \operatorname{arccsc}(c x))}{d^2 \sqrt{e x^2 + d}} - \frac{b c \sqrt{c^2 x^2 - 1} \sqrt{e x^2 + d}}{d^2 \sqrt{c^2 x^2}} + \frac{b c^2 x \operatorname{EllipticE}\left(c x, \sqrt{-\frac{e}{c^2 d}}\right) \sqrt{-c^2 x^2 + 1} \sqrt{e x^2 + d}}{d^2 \sqrt{c^2 x^2} \sqrt{c^2 x^2 - 1} \sqrt{1 + \frac{e x^2}{d}}}$$

$$- \frac{b (c^2 d + 2 e) x \operatorname{EllipticF}\left(c x, \sqrt{-\frac{e}{c^2 d}}\right) \sqrt{-c^2 x^2 + 1} \sqrt{1 + \frac{e x^2}{d}}}{d^2 \sqrt{c^2 x^2} \sqrt{c^2 x^2 - 1} \sqrt{e x^2 + d}}$$

Result(type 8, 23 leaves):

$$\int \frac{a + b \operatorname{arccsc}(c x)}{x^2 (e x^2 + d)^{3/2}} \, dx$$

Problem 46: Unable to integrate problem.

$$\int \frac{x^5 (a + b \operatorname{arccsc}(c x))}{(e x^2 + d)^{5/2}} \, dx$$

Optimal(type 3, 205 leaves, 10 steps):

$$-\frac{d^2 (a + b \operatorname{arccsc}(c x))}{3 e^3 (e x^2 + d)^{3/2}} + \frac{b x \operatorname{arctanh}\left(\frac{\sqrt{e} \sqrt{c^2 x^2 - 1}}{c \sqrt{e x^2 + d}}\right)}{e^{5/2} \sqrt{c^2 x^2}} - \frac{8 b c x \arctan\left(\frac{\sqrt{e x^2 + d}}{\sqrt{d} \sqrt{c^2 x^2 - 1}}\right) \sqrt{d}}{3 e^3 \sqrt{c^2 x^2}} + \frac{2 d (a + b \operatorname{arccsc}(c x))}{e^3 \sqrt{e x^2 + d}}$$

$$+\frac{b c d x \sqrt{c^2 x^2-1}}{3 e^2 (c^2 d+e) \sqrt{c^2 x^2} \sqrt{e x^2+d}}+\frac{(a+b \operatorname{arccsc}(c x)) \sqrt{e x^2+d}}{e^3}$$

Result(type 8, 23 leaves):

$$\int \frac{x^5 (a+b \operatorname{arccsc}(c x))}{(e x^2+d)^{5/2}}\, dx$$

Problem 47: Unable to integrate problem.

$$\int (f x)^m (e x^2+d)^3 (a+b \operatorname{arccsc}(c x))\, dx$$

Optimal(type 5, 563 leaves, 6 steps):

$$\frac{d^3 (f x)^{1+m} (a+b \operatorname{arccsc}(c x))}{f (1+m)}+\frac{3 d^2 e (f x)^{3+m} (a+b \operatorname{arccsc}(c x))}{f^3 (3+m)}+\frac{3 d e^2 (f x)^{5+m} (a+b \operatorname{arccsc}(c x))}{f^5 (5+m)}+\frac{e^3 (f x)^{7+m} (a+b \operatorname{arccsc}(c x))}{f^7 (7+m)}$$

$$+\frac{1}{c^5 f (1+m) (2+m) (4+m) (6+m) \sqrt{c^2 x^2} \sqrt{c^2 x^2-1}}\Bigg(b \Bigg(\frac{c^6 d^3 (2+m) (4+m) (6+m)}{1+m}$$

$$+\frac{e (1+m) \left(e^2 (m^2+8 m+15)^2+3 c^2 d e (3+m)^2 (m^2+13 m+42)+3 c^4 d^2 (m^4+22 m^3+179 m^2+638 m+840)\right)}{(3+m) (5+m) (7+m)}\Bigg)$$

$$x (f x)^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}+\frac{m}{2}\right], \left[\frac{3}{2}+\frac{m}{2}\right], c^2 x^2\right) \sqrt{-c^2 x^2+1}\Bigg)$$

$$+\frac{b e \left(e^2 (m^2+8 m+15)^2+3 c^2 d e (3+m)^2 (m^2+13 m+42)+3 c^4 d^2 (m^4+22 m^3+179 m^2+638 m+840)\right) x (f x)^{1+m} \sqrt{c^2 x^2-1}}{c^5 f (2+m) (3+m) (4+m) (5+m) (6+m) (7+m) \sqrt{c^2 x^2}}$$

$$+\frac{b e^2 \left(e (5+m)^2+3 c^2 d (m^2+13 m+42)\right) x (f x)^{3+m} \sqrt{c^2 x^2-1}}{c^3 f^3 (4+m) (5+m) (6+m) (7+m) \sqrt{c^2 x^2}}+\frac{b e^3 x (f x)^{5+m} \sqrt{c^2 x^2-1}}{c f^5 (6+m) (7+m) \sqrt{c^2 x^2}}$$

Result(type 8, 25 leaves):

$$\int (f x)^m (e x^2+d)^3 (a+b \operatorname{arccsc}(c x))\, dx$$

Problem 48: Unable to integrate problem.

$$\int (f x)^m (e x^2+d) (a+b \operatorname{arccsc}(c x))\, dx$$

Optimal(type 5, 201 leaves, 5 steps):

$$\frac{d (f x)^{1+m} (a+b \operatorname{arccsc}(c x))}{f (1+m)}+\frac{e (f x)^{3+m} (a+b \operatorname{arccsc}(c x))}{f^3 (3+m)}$$

$$+\frac{b\left(e(1+m)^2+c^2 d(2+m)(3+m)\right)x(fx)^{1+m}\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],c^2x^2\right)\sqrt{-c^2x^2+1}}{cf(1+m)^2(2+m)(3+m)\sqrt{c^2x^2}\sqrt{c^2x^2-1}}+\frac{bex(fx)^{1+m}\sqrt{c^2x^2-1}}{cf\left(m^2+5m+6\right)\sqrt{c^2x^2}}$$

Result(type 8, 23 leaves):

$$\int (fx)^m\left(ex^2+d\right)(a+b\,\mathrm{arccsc}(cx))\,dx$$

Test results for the 17 problems in "5.6.2 Inverse cosecant functions.txt"

Problem 1: Unable to integrate problem.

$$\int \frac{\mathrm{arccsc}(x^5 a)}{x}\,dx$$

Optimal(type 4, 78 leaves, 7 steps):

$$\frac{I\,\mathrm{arccsc}(x^5a)^2}{10}-\frac{\mathrm{arccsc}(x^5a)\ln\left(1-\left(\frac{I}{x^5a}+\sqrt{1-\frac{1}{x^{10}a^2}}\right)^2\right)}{5}+\frac{I\,\mathrm{polylog}\left(2,\left(\frac{I}{x^5a}+\sqrt{1-\frac{1}{x^{10}a^2}}\right)^2\right)}{10}$$

Result(type 8, 12 leaves):

$$\int \frac{\mathrm{arccsc}(x^5 a)}{x}\,dx$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int x\,\mathrm{arccsc}\left(\frac{a}{x}\right)dx$$

Optimal(type 3, 39 leaves, 4 steps):

$$-\frac{a^2\arcsin\left(\frac{x}{a}\right)}{4}+\frac{x^2\arcsin\left(\frac{x}{a}\right)}{2}+\frac{ax\sqrt{1-\frac{x^2}{a^2}}}{4}$$

Result(type 3, 92 leaves):

$$\frac{x^2\,\mathrm{arccsc}\left(\frac{a}{x}\right)}{2}-\frac{a\sqrt{\frac{a^2}{x^2}-1}\;x\arctan\left(\frac{1}{\sqrt{\frac{a^2}{x^2}-1}}\right)}{4\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}}+\frac{\left(\frac{a^2}{x^2}-1\right)x^3}{4a\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int x^4 \operatorname{arccsc}(b x + a) \, dx$$

Optimal(type 3, 173 leaves, 9 steps):

$$\frac{a^5 \operatorname{arccsc}(b x + a)}{5 b^5} + \frac{x^5 \operatorname{arccsc}(b x + a)}{5} + \frac{(40 a^4 + 40 a^2 + 3) \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{(b x + a)^2}}\right)}{40 b^5} - \frac{a (53 a^2 + 20) (b x + a) \sqrt{1 - \frac{1}{(b x + a)^2}}}{30 b^5} - \frac{11 a x^2 (b x + a) \sqrt{1 - \frac{1}{(b x + a)^2}}}{60 b^3} + \frac{x^3 (b x + a) \sqrt{1 - \frac{1}{(b x + a)^2}}}{20 b^2} + \frac{(58 a^2 + 9) (b x + a)^2 \sqrt{1 - \frac{1}{(b x + a)^2}}}{120 b^5}$$

Result(type 3, 506 leaves):

$$\frac{((b x + a)^2 - 1) x^3}{20 b^2 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}} (b x + a)} + \frac{\sqrt{(b x + a)^2 - 1}\, a^5 \arctan\left(\frac{1}{\sqrt{(b x + a)^2 - 1}}\right)}{5 b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}} (b x + a)} + \frac{\sqrt{(b x + a)^2 - 1}\, a^4 \ln\left(b x + a + \sqrt{(b x + a)^2 - 1}\right)}{b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}} (b x + a)} + \frac{3 ((b x + a)^2 - 1) x}{40 b^4 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}} (b x + a)} + \frac{\sqrt{(b x + a)^2 - 1}\, a^2 \ln\left(b x + a + \sqrt{(b x + a)^2 - 1}\right)}{b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}} (b x + a)} + \frac{x^5 \operatorname{arccsc}(b x + a)}{5} - \frac{77 ((b x + a)^2 - 1) a^3}{60 b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}} (b x + a)} - \frac{71 ((b x + a)^2 - 1) a}{120 b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}} (b x + a)} + \frac{3 \sqrt{(b x + a)^2 - 1} \ln\left(b x + a + \sqrt{(b x + a)^2 - 1}\right)}{40 b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}} (b x + a)} - \frac{11 ((b x + a)^2 - 1) x^2 a}{60 b^3 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}} (b x + a)} + \frac{29 ((b x + a)^2 - 1) x a^2}{60 b^4 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}} (b x + a)}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int x^3 \operatorname{arccsc}(b x + a) \, dx$$

Optimal(type 3, 135 leaves, 8 steps):

$$-\frac{a^4 \operatorname{arccsc}(b x + a)}{4 b^4} + \frac{x^4 \operatorname{arccsc}(b x + a)}{4} - \frac{a (2 a^2 + 1) \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{(b x + a)^2}}\right)}{2 b^4} + \frac{(17 a^2 + 2) (b x + a) \sqrt{1 - \frac{1}{(b x + a)^2}}}{12 b^4}$$

$$+\frac{x^2 (bx+a)\sqrt{1-\frac{1}{(bx+a)^2}}}{12 b^2}-\frac{a (bx+a)^2\sqrt{1-\frac{1}{(bx+a)^2}}}{3 b^4}$$

Result(type 3, 359 leaves):

$$\frac{x^4 \operatorname{arccsc}(bx+a)}{4}+\frac{((bx+a)^2-1)\, x^2}{12 b^2\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}-\frac{\sqrt{(bx+a)^2-1}\; a^4 \arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)}{4 b^4\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}-\frac{((bx+a)^2-1)\, x a}{3 b^3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}$$
$$-\frac{\sqrt{(bx+a)^2-1}\; a^3 \ln\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{b^4\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}+\frac{13\, ((bx+a)^2-1)\, a^2}{12 b^4\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}-\frac{\sqrt{(bx+a)^2-1}\; a \ln\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{2 b^4\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}$$
$$+\frac{(bx+a)^2-1}{6 b^4\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{arccsc}(bx+a)\, dx$$

Optimal(type 3, 100 leaves, 7 steps):

$$\frac{a^3 \operatorname{arccsc}(bx+a)}{3 b^3}+\frac{x^3 \operatorname{arccsc}(bx+a)}{3}+\frac{(6 a^2+1)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{6 b^3}-\frac{5 a (bx+a)\sqrt{1-\frac{1}{(bx+a)^2}}}{6 b^3}$$
$$+\frac{x (bx+a)\sqrt{1-\frac{1}{(bx+a)^2}}}{6 b^2}$$

Result(type 3, 271 leaves):

$$\frac{x^3 \operatorname{arccsc}(bx+a)}{3}+\frac{\sqrt{(bx+a)^2-1}\; a^3 \arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)}{3 b^3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}+\frac{((bx+a)^2-1)\, x}{6 b^2\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}$$
$$+\frac{\sqrt{(bx+a)^2-1}\; a^2 \ln\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{b^3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}-\frac{5\, ((bx+a)^2-1)\, a}{6 b^3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}+\frac{\sqrt{(bx+a)^2-1}\; \ln\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{6 b^3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{arccsc}(b x + a)}{x^2}\, dx$$

Optimal(type 3, 63 leaves, 6 steps):

$$-\frac{b \operatorname{arccsc}(b x + a)}{a} - \frac{\operatorname{arccsc}(b x + a)}{x} - \frac{2 b \arctan\left(\frac{a - \tan\left(\frac{\operatorname{arccsc}(b x + a)}{2}\right)}{\sqrt{-a^2 + 1}}\right)}{a \sqrt{-a^2 + 1}}$$

Result(type 3, 153 leaves):

$$-\frac{\operatorname{arccsc}(b x + a)}{x} - \frac{b \sqrt{(b x + a)^2 - 1}\, \arctan\left(\frac{1}{\sqrt{(b x + a)^2 - 1}}\right)}{\sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)\, a} + \frac{b \sqrt{(b x + a)^2 - 1}\, \ln\left(\frac{2 \left(\sqrt{a^2 - 1}\, \sqrt{(b x + a)^2 - 1} + a (b x + a) - 1\right)}{b x}\right)}{\sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)\, a \sqrt{a^2 - 1}}$$

Problem 13: Unable to integrate problem.

$$\int x^{-1 + n} \operatorname{arccsc}\left(a + b x^n\right) dx$$

Optimal(type 3, 46 leaves, 6 steps):

$$\frac{\left(a + b x^n\right) \operatorname{arccsc}\left(a + b x^n\right)}{b n} + \frac{\operatorname{arctanh}\left(\sqrt{1 - \frac{1}{\left(a + b x^n\right)^2}}\right)}{b n}$$

Result(type 8, 16 leaves):

$$\int x^{-1 + n} \operatorname{arccsc}\left(a + b x^n\right) dx$$

Problem 14: Unable to integrate problem.

$$\int e^{\operatorname{arccsc}(a x)} x^2\, dx$$

Optimal(type 5, 101 leaves, 6 steps):

$$\frac{\left(\frac{4}{5} - \frac{12 I}{5}\right) e^{(1 + 3 I) \operatorname{arccsc}(a x)} \operatorname{hypergeom}\left(\left[3, \frac{3}{2} - \frac{I}{2}\right], \left[\frac{5}{2} - \frac{I}{2}\right], \left(\frac{I}{a x} + \sqrt{1 - \frac{1}{x^2 a^2}}\right)^2\right)}{a^3}$$
$$+ \frac{\left(-\frac{8}{5} + \frac{24 I}{5}\right) e^{(1 + 3 I) \operatorname{arccsc}(a x)} \operatorname{hypergeom}\left(\left[4, \frac{3}{2} - \frac{I}{2}\right], \left[\frac{5}{2} - \frac{I}{2}\right], \left(\frac{I}{a x} + \sqrt{1 - \frac{1}{x^2 a^2}}\right)^2\right)}{a^3}$$

Result(type 8, 11 leaves):

$$\int e^{\operatorname{arccsc}(a x)} x^2 \, dx$$

Problem 15: Unable to integrate problem.

$$\int \frac{e^{\operatorname{arccsc}(a x)}}{x^2} \, dx$$

Optimal(type 3, 31 leaves, 3 steps):

$$-\frac{e^{\operatorname{arccsc}(a x)}}{2 x} - \frac{a e^{\operatorname{arccsc}(a x)} \sqrt{1 - \frac{1}{x^2 a^2}}}{2}$$

Result(type 8, 11 leaves):

$$\int \frac{e^{\operatorname{arccsc}(a x)}}{x^2} \, dx$$

Problem 16: Unable to integrate problem.

$$\int \frac{e^{\operatorname{arccsc}(a x)}}{x^4} \, dx$$

Optimal(type 3, 70 leaves, 6 steps):

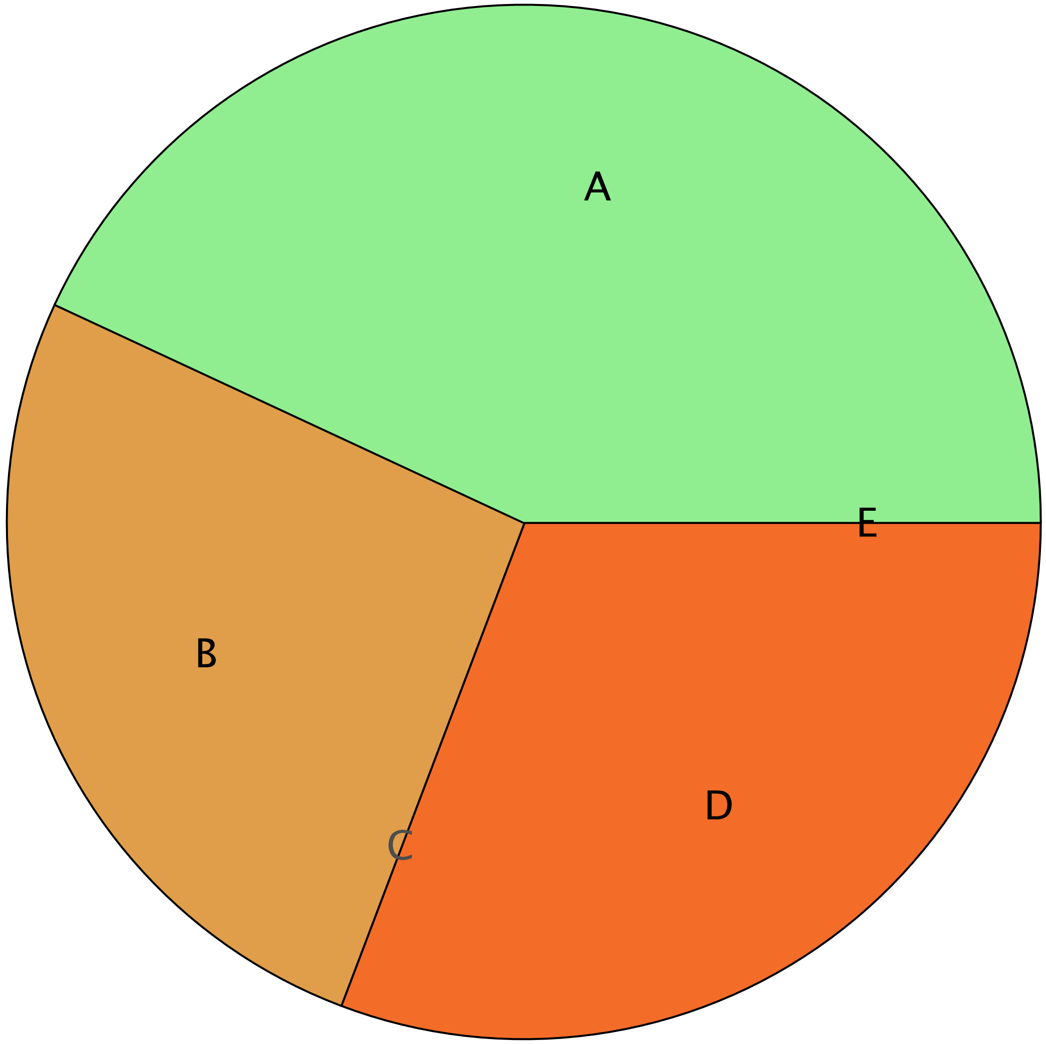
$$-\frac{a^2 e^{\operatorname{arccsc}(a x)}}{8 x} + \frac{a^3 e^{\operatorname{arccsc}(a x)} \cos(3 \operatorname{arccsc}(a x))}{40} + \frac{3 a^3 e^{\operatorname{arccsc}(a x)} \sin(3 \operatorname{arccsc}(a x))}{40} - \frac{a^3 e^{\operatorname{arccsc}(a x)} \sqrt{1 - \frac{1}{x^2 a^2}}}{8}$$

Result(type 8, 11 leaves):

$$\int \frac{e^{\operatorname{arccsc}(a x)}}{x^4} \, dx$$

Summary of Integration Test Results

65 integration problems



A - 28 optimal antiderivatives
B - 17 more than twice size of optimal antiderivatives
C - 0 unnecessarily complex antiderivatives
D - 20 unable to integrate problems
E - 0 integration timeouts